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# In search of Dirac neutrino masses through baryogenesis in normal hierarchy

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**Abstract.** We point out a possible way to settle the issue of the Dirac neutrino mass hierarchy. Constraining the observed baryon asymmetry to the normal hierarchy mass model within the seesaw framework, we look for the possible structure of coveted Dirac neutrino masses. We have found the possible structure of the Dirac neutrino masses to be $(\lambda^7, \lambda^2, 1)v$ in terms of the parameter $\lambda = 0.3$, with $v$ as an overall scale factor.

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The canonical seesaw formula [1–3] relates the light left handed Majorana neutrino mass matrix $m_{\mathrm{LL}}$, the Dirac neutrino mass matrix $m_{\mathrm{LR}}$ and the right handed heavy Majorana mass matrix $M_{\mathrm{RR}}$ in an elegant way:

$$m_{\mathrm{LL}} = -m_{\mathrm{LR}} M_{\mathrm{RR}}^{-1} m_{\mathrm{LR}}^{\mathrm{T}} . \qquad (1)$$

For a specific structure of $m_{\mathrm{LL}}$ we can have different choices of $m_{\mathrm{LR}}$ and $M_{\mathrm{RR}}$. The choice of diagonal $m_{\mathrm{LR}}$ and non-diagonal $M_{\mathrm{RR}}$ plays an important role in predicting the neutrino masses and mixings through $m_{\mathrm{LL}}$, whereas the choice of a non-diagonal $m_{\mathrm{LR}}$ and a diagonal $M_{\mathrm{RR}}$ plays a crucial role in the prediction of the baryon asymmetry [4] via lepton asymmetry [5, 6] produced by the decay of lightest of the heavy Majorana neutrino $M_1$.

At present we do not have specific information concerning the structure of the Dirac neutrino mass matrices. Fortunately some grand unified theory such as SO(10) GUT has predicted the possible structure [7] of $m_{\mathrm{LR}} = \mathrm{diag}(\lambda^m, \lambda^n, 1)v$ in terms of the Wolfenstein parameter $\lambda$ ($\simeq 0.22$), where $v$ is the overall scale factor. The $(m, n)$ pair can have three variations: $(m, n) \equiv (8, 4)$, $(m, n) \equiv (4, 2)$, $(m, n) \equiv (6, 2)$ respectively for the up quark, down quark and charge lepton mass hierarchy.

With its underlying beauty the seesaw formula itself cannot discriminate the above three choices of the Dirac neutrino mass hierarchy and the problem persist even in its non-canonical extension.

To understand the Dirac neutrino masses in the light of the observed baryon asymmetry we consider the normal hierarchical mass model [8–10], which corresponds to good predictions of neutrino masses, mixings and baryon asymmetry within a unified framework, and this model is also stable under radiative corrections in the minimal supersymmetric standard model. For this model [8–10]

$$M_{\mathrm{RR}} = \begin{pmatrix} \lambda^{2m-1} & \lambda^{m+n-1} & \lambda^{m-1} \\ \lambda^{m+n-1} & \lambda^{m+n-2} & 0 \\ \lambda^{m-1} & 0 & 1 \end{pmatrix} v_0 ,$$

$$-m_{\mathrm{LL}} = \begin{pmatrix} -\lambda^4 & \lambda & \lambda^3 \\ \lambda & 1-\lambda & -1 \\ \lambda^3 & -1 & 1-\lambda^3 \end{pmatrix} m_0 .$$

The input values are $\lambda = 0.3$, $m_0 = 0.03\,\mathrm{eV}$ and $v_0 = v^2/m_0 = 1.01 \times 10^{15}$ GeV for $v = 174$ GeV. The value of the parameter $\lambda = 0.3$ is within the range [7], i.e., $0.16 \leq \lambda \leq 0.37$.

The above $m_{\mathrm{LL}}$ leads to correct neutrino mass parameter and mixing angles: $\Delta m_{21}^2 = 9.04 \times 10^{-5}\,\mathrm{eV}^2$, $\Delta m_{23}^2 = 3.01 \times 10^{-3}\,\mathrm{eV}^2$, $\tan^2\theta_{12} = 0.55$, $\sin^2 2\theta_{23} = 0.98$, $\sin\theta_{13} = 0.074$. These values are almost consistent with recent data [8–10].

For the normal hierarchical mass structure of Majorana neutrinos, considering the out of equilibrium and $CP$-vio-

**Table 1.** The three right-handed Majorana neutrino masses $M_j$ $(j = 1, 2, 3)$ in GeV for different choices of $(m, n)$ pair

| $(m,n)$ | $M_1$ | $M_2$ | $M_3$ |
|---|---|---|---|
| $(4,2)$ | $1.16 \times 10^{12}$ | $8.83 \times 10^{12}$ | $1.01 \times 10^{15}$ |
| $(6,2)$ | $6.51 \times 10^{10}$ | $7.97 \times 10^{11}$ | $1.01 \times 10^{15}$ |
| $(7,2)$ | $1.87 \times 10^{10}$ | $2.39 \times 10^{11}$ | $1.01 \times 10^{15}$ |
| $(8,4)$ | $5.27 \times 10^{8}$ | $6.45 \times 10^{9}$ | $1.01 \times 10^{15}$ |

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**Table 2.** The values of $M_1$ in GeV, $\varepsilon_1$ and $Y_{\mathrm{B}}^{\mathrm{SM}}$ for different choices of the $(m,n)$ pair

| $(m,n)$ | $(8,4)$ | $(6,2)$ | $(4,2)$ | $(7,2)$ |
|---|---|---|---|---|
| $M_1$ (GeV) | $5.27\times10^{8}$ | $6.51\times10^{10}$ | $1.16\times10^{12}$ | $1.87\times10^{10}$ |
| $\varepsilon_1$ | $4.78\times10^{-9}$ | $5.9\ \times10^{-7}$ | $1\ \ \times10^{-5}$ | $1.7\ \times10^{-7}$ |
| $Y_{\mathrm{B}}^{\mathrm{SM}}$ | $1.76\times10^{-11}$ | $2.17\times10^{-9}$ | $3.24\times10^{-8}$ | $6.24\times10^{-10}$ |

lating decay of the physical Majorana neutrino $M_1$, the $CP$ asymmetry is expressed as [11–13]

$$\varepsilon_1 = \frac{3M_1}{16\pi v^2}\frac{\mathrm{Im}\left[\left(h^* m_{\mathrm{LL}}^{I} h^\dagger\right)_{11}\right]}{(hh^\dagger)_{11}}, \tag{2}$$

where $h = m_{\mathrm{LR}}/v$ is the $3\times3$ Yukawa coupling matrix normalised to $h_{33}=1$. Again the electroweak sphaleron interaction [5, 6, 14] partly converts the lepton asymmetry to baryon asymmetry. The baryon asymmetry of the universe $Y_{\mathrm{B}}^{\mathrm{SM}}$ which is defined as the ratio of baryon number density $(n_{\mathrm{B}})$ to photon number density $(n_\gamma)$ in the standard model (SM) case is expressed in terms of the washout factor $\kappa_1$ and $\varepsilon_1$ as [15]

$$Y_{\mathrm{B}}^{\mathrm{SM}} = \frac{n_{\mathrm{B}}}{n_\gamma} \simeq 0.0216\kappa_1\varepsilon_1\,. \tag{3}$$

The washout factor $\kappa_1 = (2\sqrt{K^2+9})^{-1}$ for $0<K<10$ is expressed in terms of $K = \frac{\tilde{m}}{m^*}$, where

$$\tilde{m} = \frac{(hh^\dagger)_{11}v^2}{M_1}$$

is the effective neutrino mass and

$$m^* = \frac{16\pi^{5/2}}{3\sqrt{5}}g^*\frac{1}{2}\frac{v^2}{M_{\mathrm{pl}}}$$

is the equilibrium neutrino mass. In the standard model scenario $g^* = 106.75$ is the value of the degree of freedom and $M_{\mathrm{pl}} = 1.2\times10^{19}$ GeV. Out of equilibrium decay of $M_1$ is characterised by $K<1$. Turning to the baryogenesis scenario for the normal hierarchy case, we choose a basis $U_{\mathrm{R}}$ where $M_{\mathrm{RR}}^{\mathrm{diag}} = U_{\mathrm{R}}^{\mathrm{T}} M_{\mathrm{RR}} U_{\mathrm{R}} = \mathrm{diag}(M_1, M_2, M_3)$ with real and positive eigenvalues [16]. We transform $m_{\mathrm{LR}} = \mathrm{diag}(\lambda^m, \lambda^n, 1)v$ to the $U_{\mathrm{R}}$ basis by $m_{\mathrm{LR}} \to m'_{\mathrm{LR}} = m_{\mathrm{LR}}U_{\mathrm{R}}$. In this prime basis the Yukawa coupling becomes $h = (m_{\mathrm{LR}}U_{\mathrm{R}})/v$.

For the particular choice $(m,n) = (7,2)$ we have $M_1 = 1.87\times10^{10}$ GeV, $(hh^\dagger)_{11} \simeq 4.78\times10^{-8}$, $\mathrm{Im}(h^* m_{\mathrm{LL}} h^\dagger)_{11} \simeq 2.21\times10^{-19}$, $\tilde{m} \simeq 7.74\times10^{-5}$ eV, $m^* \simeq 1.08\times10^{-3}$ eV, $K = 0.072$ and $\kappa = 0.17$. This leads to $\varepsilon_1 \simeq 1.7\times10^{-7}$ and $Y_{\mathrm{B}}^{\mathrm{SM}} = 6.24\times10^{-10}$, exactly within the experimental bound [17]: $Y_{\mathrm{B}}^{\mathrm{CMB}} = (6.1^{+0.3}_{-0.2})\times10^{-10}$. Tables 1 and 2 contain the eigenvalues and results for different choices of the $(m,n)$ pair.

For the hierarchical structure of the heavy Majorana neutrino masses $M_1 > 4\times10^{8}$ GeV agrees with the famous Davidson–Ibarra bound [18] . Assuming the normal hierarchical mass model as a favourable choice of nature we have found the Dirac neutrino masses to be $(\lambda^7, \lambda^2, 1)v$ with $\lambda = 0.3$ and $v$ as the overall scale factor.

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## References

1. M. Gell-Mann, P. Ramond, R. Slansky, in: Proc. Workshop Supergravity, Stony Brook, New York, 1979, ed. by P. Van Nieuwenhuizen, D. Freedman (North-Holland, Amsterdam, 1979)
2. T. Yanagida, KEK Lectures, 1979, unpublished
3. R.N. Mahapatra, G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980)
4. A.D. Sakharov, JETP Lett. **5**, 24 (1967)
5. V.A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985)
6. M. Fukugita, T. Yanagida, Phys. Lett. B **174**, 45 (1986)
7. P.M. Fishbane, P. Kaus, J. Phys. G **25**, 1629 (1999)
8. N. Nimai Singh, M. Patgiri, Int. J. Mod. Phys. A **17**, 3629 (2002)
9. M. Patgiri, N. Nimai Singh, Int. J. Mod. Phys. A **18**, 443 (2003)
10. N. Nimai Singh, M. Patgiri, M.K. Das, Pramana J. Phys. **66**, 361 (2006)
11. N. Sahu, S.U. Sankar, Phys. Rev. D **71**, 013006 (2005) [hep-ph/0406065]
12. T. Hambye, G. Senjanovic, Phys. Lett. B **582**, 73 (2004) [hep-ph/0307237]
13. S. Antush, S.F. King, Phys. Lett. B **597**, 199 (2004) [hep-ph/0405093]
14. E.W. Kolb, M.S. Turner, The Early Universe (Addison-Wesley, New York, 1990)
15. C.H. Albright, S.M. Barr, Phys. Rev. D **70**, 033013 (2004) [hep-ph/0404095]
16. K.S. Babu, A. Bachri, Nucl. Phys. B **738**, 76 (2006) [hep-ph/0509091]
17. D.N. Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003) [astro-ph/0302209]
18. S. Davidson, A. Ibarra, Phys. Lett. B **535**, 25 (2002)